

# Mathematics and Units of Measure

## NUMBER NOTATION

The decimal number system uses base 10. Place value:

123,456,789,012

Each section shows hundreds, tens, and ones.

Expanded notation:

$$6,824 = 6 \times 10^3 + 8 \times 10^2 + 2 \times 10^1 + 4 \times 10^0$$

Scientific notation:  $6,800 = 6.8 \times 10^3$

## SYMBOLS

- < Is less than
- > Is greater than
- = Is equal to
- ≈ Is approximate to
- ≤ Is less than or equal to
- ≥ Is greater than or equal to

## ORDER OF OPERATIONS

- P Do operations within Parentheses ( ) and other grouping symbols, like absolute value and square root.
- E Do Exponents and roots.
- MD Do Multiplication x and Division ÷ in order from left to right.
- AS Do Addition + and Subtraction - in order from left to right.

## FRACTIONS, DECIMALS, PERCENTAGES

- 3 - numerator
- 5 - denominator
- To add or subtract fractions, first obtain a common denominator:
- $\frac{1}{3} + \frac{2}{5} = \frac{5}{15} + \frac{6}{15} = \frac{11}{15}$
- To multiply:
- $\frac{1}{3} \times \frac{2}{5} = \frac{1 \times 2}{3 \times 5} = \frac{2}{15}$
- To divide, multiply the first fraction by the reciprocal of the second fraction.
- $\frac{2}{3} \div \frac{1}{6} = \frac{2}{3} \times \frac{6}{1} = \frac{12}{3} = 4$

- 1 = 10 = 100%
- 3/4 = 0.75 = 75%
- 2/3 = 0.666... = 66.6%
- 1/2 = 0.5 = 50%
- 1/3 = 0.333... = 33.3%
- 1/4 = 0.25 = 25%
- 1/5 = 0.2 = 20%
- 1/6 = 0.166... = 16.6%
- 1/8 = 0.125 = 12.5%
- 1/9 = 0.111... = 11.1%
- 1/10 = 0.1 = 10%
- 1/12 = 0.083... = 8.3%

## MEASUREMENTS AND CONVERSIONS

1000	100	10	1	1	01	001
KILO	HECTO	DECA	DECI	CENTI	MILLI	
km	hm	dam	m	dm	cm	mm
kg	hg	dag	g	dg	cg	mg
kl	hL	daL	L	dL	cl	mL

Length

1 foot (ft.)	=	12 inches (in.)
1 yard (yd.)	=	3 feet
1 mile (mi.)	=	1,760 yards
	=	5,280 feet

Capacity

1 tablespoon (Tbsp.)	=	3 teaspoons (tsp.)
1 cup (c.)	=	8 floz.
1 pint (pt.)	=	2 c.
1 quart (qt.)	=	2 pt. = 4 c. = 32 floz.
1 gallon (gal.)	=	4 qt.

Area

1 ft. <sup>2</sup>	=	144 in. <sup>2</sup>
1 yd. <sup>2</sup>	=	9 ft. <sup>2</sup>
1 acre	=	4,840 yd. <sup>2</sup>
1 m <sup>2</sup>	=	10,000 cm <sup>2</sup>
1 hectare (ha)	=	10,000 m <sup>2</sup>
1 km <sup>2</sup>	=	100 ha

Mass

1 pound (lb.)	=	16 ounces (oz.)
1 ton (tn.)	=	2,000 lbs.
1 kg	=	1,000 g
1 metric tonne (t)	=	1,000 kg

Length/Area

to go from	to	multiply by	
cm	→	in.	0.39
in.	→	cm	2.54
m	→	ft.	3.28
ft.	→	m	0.30
km	→	mi.	0.62
mi.	→	km	1.61
m <sup>2</sup>	→	ft. <sup>2</sup>	10.76
ft. <sup>2</sup>	→	m <sup>2</sup>	0.09
km <sup>2</sup>	→	mi. <sup>2</sup>	0.39
mi. <sup>2</sup>	→	km <sup>2</sup>	2.59

Weight/Capacity

to go from	to	multiply by	
g	→	oz.	0.0353
oz.	→	g	28.35
kg	→	lbs.	2.2046
lbs.	→	kg	0.4536
t	→	tn.	1.1023
tn.	→	t	0.9072
mL	→	fLoz.	0.0338
fL oz.	→	mL	29.574
L	→	US gal.	0.2642
US gal.	→	L	3.785

## SQUARES AND SQUARE ROOTS

n	n <sup>2</sup>	√n
1	1	1
2	4	1.414
3	9	1.732
4	16	2
5	25	2.236
6	36	2.449
7	49	2.646
8	64	2.828
9	81	3
10	100	3.162
12	144	3.464
15	225	3.873
20	400	4.472
25	625	5
100	10,000	10
1/2	1/4	0.707
1/4	1/16	1/2

## COMMON UNITS

used with the International System

MEASUREMENT	ABBREV.	RELATION
meter	m	length
hectare	ha	area
kelvin	K	thermodynamic temp.
kilogram	kg	mass
liter	L	volume or capacity
second	s	time
hertz	Hz	frequency
degree Celsius	°C	temperature
joule	J	energy, work
pascal	Pa	pressure, stress
newton	N	force
watt	W	power, radiant flux
ampere	A	electric current
volt	V	electric potential

\*Canadian preferred spelling: metre, litre.

## TEMPERATURE

°C = 5/9 (°F - 32)  
°F = 9/5 °C + 32

# Properties of Addition and Multiplication



Commutative property of addition  
 $a + b = b + a$

Commutative property of multiplication  
 $ab = ba$

Associative property of addition  
 $a + (b + c) = (a + b) + c$

Associative property of multiplication  
 $a(bc) = (abc)$

Distributive property of multiplication over addition  
 $a(b + c) = ab + ac$

Distributive property of multiplication over subtraction  
 $a(b - c) = ab - ac$

## MULTIPLICATION CHART

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38
3	3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	48	51	54	57
4	4	8	12	16	20	24	28	32	36	40	44	48	52	56	60	64	68	72	76
5	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95
6	6	12	18	24	30	36	42	48	54	60	66	72	78	84	90	96	102	108	114
7	7	14	21	28	35	42	49	56	63	70	77	84	91	98	105	112	119	126	133
8	8	16	24	32	40	48	56	64	72	80	88	96	104	112	120	128	136	144	152
9	9	18	27	36	45	54	63	72	81	90	99	108	117	126	135	144	153	162	171
10	10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190
11	11	22	33	44	55	66	77	88	99	110	121	132	143	154	165	176	187	198	209
12	12	24	36	48	60	72	84	96	108	120	132	144	156	168	180	192	204	216	228

# Algebra

## EXPANDING

$a(b + c) = ab + ac$   
 $(a - b)^2 = a^2 - 2ab + b^2$   
 $(a + b)^2 = a^2 + 2ab + b^2$   
 $(a + b)(c + d) = ac + ad + bc + bd$   
 $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$   
 $(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

## FACTORIZING

$a^2 - b^2 = (a + b)(a - b)$   
 $a^3 - ab^2 = ab(a + 1)(a - 1)$   
 $a^3 + 2ab + b^3 = (a + b)^2$   
 $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$   
 $a^3 - 2ab + b^3 = (a - b)^2$   
 $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

## ROOTS OF A QUADRATIC

The solution for a quadratic equation in the form of  $ax^2 + bx + c = 0$  can be found by using the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

## LAW OF EXPONENTS

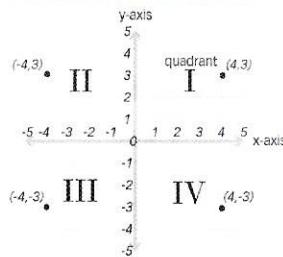
If  $a, b \in \mathbb{R}, a, b \geq 0$  and  $p, q, r, s$  are  $\in \mathbb{Q}$ , then:

- $a^p a^q = a^{p+q}$
- $\frac{a^p}{a^q} = a^{p-q}$  ( $a \neq 0$ )
- $(a^p)^q = a^{pq}$
- $(ab)^p = a^p b^p$
- $\left(\frac{a}{b}\right)^p = \frac{a^p}{b^p}$  ( $b \neq 0$ )
- $a^0 = 1$  ( $a \neq 0$ )
- $a^{-p} = \frac{1}{a^p}$  ( $a \neq 0$ )
- $a^p = \sqrt[p]{a^p}$

## LOGARITHMS

$\log_a x = n \iff x = a^n$  (log to the base a)  
 $\log_a(xy) = \log_a x + \log_a y$   
 $\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$   
 $\log_a x^p = p \log_a x$   
 $\log_a a^x = x$   
 $a^{\log_a x} = x$   
 $\log_a x = \log_a x$   
 $\log_a a$

## THE COORDINATE PLANE



Common log: A log that is written without a base:  $\log x = n$ , the base is 10:  $\log_{10} x = n$ . All  $\log_a$  rules apply.

Natural log: A log that is written in  $x = n$ , where the base is  $e$ :  $\log_e x = n$ . All  $\log_a$  rules apply.